

In this chapter, you have studied the following points:

1. Though Euclid defined a point, a line, and a plane, the definitions are not accepted by mathematicians. Therefore, these terms are now taken as undefined.
2. Axioms or postulates are the assumptions which are obvious universal truths. They are not proved.
3. Theorems are statements which are proved, using definitions, axioms, previously proved statements and deductive reasoning.

4. Some of Euclid's axioms were :

- (1) Things which are equal to the same thing are equal to one another.
- (2) If equals are added to equals, the wholes are equal.
- (3) If equals are subtracted from equals, the remainders are equal.
- (4) Things which coincide with one another are equal to one another.
- (5) The whole is greater than the part.
- (6) Things which are double of the same things are equal to one another.
- (7) Things which are halves of the same things are equal to one another.

5. Euclid's postulates were :

Postulate 1 : A straight line may be drawn from any one point to any other point.

Postulate 2 : A terminated line can be produced indefinitely.

Postulate 3 : A circle can be drawn with any centre and any radius.

Postulate 4 : All right angles are equal to one another.

Postulate 5 : If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

6. Two equivalent versions of Euclid's fifth postulate are:

- (i) 'For every line l and for every point P not lying on l , there exists a unique line m passing through P and parallel to l '.
- (ii) Two distinct intersecting lines cannot be parallel to the same line.

7. All the attempts to prove Euclid's fifth postulate using the first 4 postulates failed. But they led to the discovery of several other geometries, called non-Euclidean geometries.

Exercise 5.1

Q1. Which of the following statements are true and which are false? Give reasons for your answers.

(i) Only one line can pass through a single point.

Ans False, There can be infinite number of lines that can be drawn through a single point.

(ii) There are an infinite number of lines which pass through two distinct points.

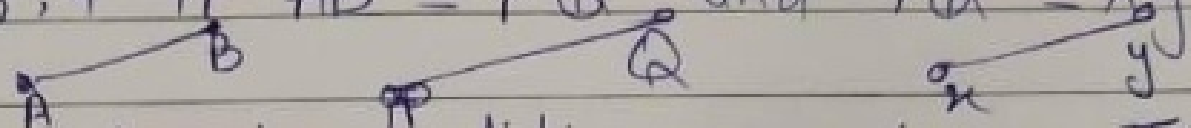
Ans False, Through two distinct points there can be only one line that can be drawn.

(iii) A terminated line can be produced indefinitely on both the sides.

Ans True, A line that is terminated can be indefinitely produced on both sides as a line can be extended on both its sides infinitely.

Q. (iv) If two circles are equal, then their radii are equal.
 Ans. True. The Radii of two Circles are equal when the two circles are equal. The Circumference and the Centre of both the circles ~~are~~ coincide; and thus, the radius of the two circles should be equal.

Q. (v) In Fig. 5.9 if $AB = PQ$ and $PQ = xy$, then $AB = xy$



Ans. True. According to Euclid's 1st axiom: - Things which are equal to the same thing are also equal to one another.

2. Give a definition for each of the following terms. Are there other terms that need to be defined first? What are they, and how might you define them?

- (i) parallel lines**
- (ii) perpendicular lines**
- (iii) line segment**
- (iv) radius of a circle**
- (v) square**

Solution:

Yes there are other terms which need to be defined first, they are:

Plane: Flat surfaces in which geometric figures can be drawn are known as plane. A plane surface is a surface which lies evenly with the straight lines on itself.

Point: A dimensionless dot which is drawn on a plane surface is known as point. A point is that which has no part.

Line: A collection of points that has only length and no breadth is known as a line. And it can be extended on both directions. A line is breadth-less length.

- (i) **Parallel lines-** Parallel lines are those lines which never intersect each other and are always at a constant distance perpendicular to each other. Parallel lines can be two or more lines.
- (ii) **Perpendicular lines-** Perpendicular lines are those lines which intersect each other in a plane at right angles then the lines are said to be perpendicular to each other.
- (iii) **Line Segment-** When a line cannot be extended any further because of its two end points then the line is known as a line segment. A line segment has 2 end points.
- (iv) **Radius of circle-** A radius of a circle is the line from any point on the circumference of the circle to the center of the circle.
- (v) **Square-** A quadrilateral in which all the four sides are said to be equal and each of its internal angle is right angles is called square.

3. Consider two 'postulates' given below:

- (i) **Given any two distinct points A and B, there exists a third point C which is in between A and B.**
- (ii) **There exist at least three points that are not on the same line.**

Do these postulates contain any undefined terms? Are these postulates consistent? Do they follow from Euclid's postulates? Explain.

Solution:

Yes, these postulates contain undefined terms. Undefined terms in the postulates are:

- There are many points that lie in a plane. But, in the postulates given here, the position of the point C is not given, as of whether it lies on the line segment joining AB or not.
- On top of that, there is no information about whether the points are in same plane or not.

Yes, these postulates are consistent when we deal with these two situation:

- Point C is lying on the line segment AB in between A and B.
- Point C does not lie on the line segment AB.

No, they don't follow from Euclid's postulates. They follow the axioms.

Friday - 31 July 2020

Ex. 5.1

If a point C lies between two points A and B such that $AC = BC$, then prove that $AC = \frac{1}{2}AB$.

Explain by drawing the figure.

Given = $AC = BC$
C is the mid point of AB

Prove = $AC = \frac{1}{2}AB$

$$AB = AC + BC$$

$$AB = AC + AC$$

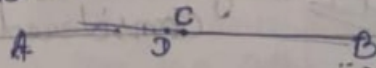
$$AB = 2AC$$

$$\frac{AB}{2} = AC$$

$$\frac{1}{2}AB = AC$$

$$AC = \frac{1}{2}AB \text{ (H.P.)}$$

Q5. In Q4, point C is called a mid point of line segment AB. Prove that every line segment has one and only one mid point.



Let the given line AB is having two mid point C' and D'.

$$AC' = \frac{1}{2}AB \text{ (1) (from Q. 4)}$$

$$\text{and } AD' = \frac{1}{2}AB \text{ (2)}$$

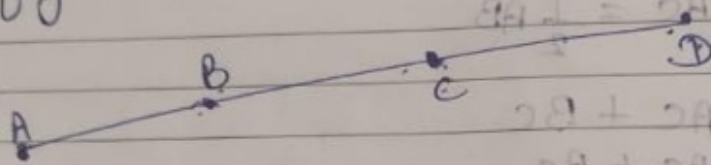
from eq (1) & (2)

$$AC = AD \text{ (Euclid's 7 axioms)}$$

It means C and D coincide

Thus, every line segment has one and only one mid point.

Q6. In fig 5.10 if $AC = BD$, then prove that $AB = CD$.



Sol. Given $AC = BD$
Prove $AB = CD$

$$AC = AB + BC$$

$$BD = BC + CD$$

$$AC = BD \text{ (given)}$$

$$AB + BC = BC + CD$$

$$AB + BC - BC = + CD$$

$$AB = CD \text{ (H.P)}$$

Q7. Why is Axiom 5, in the list of Euclid's axioms, considered a Universal truth?

Ans. Axiom 5:- Whole is always greater than its part.

True because part is include in the whole and never greater than whole. This is a universal truth.

Ex. 5.2

How would you rewrite Euclid's fifth Postulate so it would be easier to understand?

Postulate 5

When two lines are cut by a third line, such that the sum of interior angles is less than 180° on ^{one} side then first two lines intersect on the same side.

Does Euclid's fifth postulate imply the existence of parallel lines? Explain.

Yes, If a straight line l falls on two lines m and n such that sum of the interior angles on one side of l is two right angles, then by Euclid's 5th postulate, lines m and n will not meet on this side of l . Also, we know that the sum of the interior angles on the other side of the line l will be two right angles too. Thus, they will not meet on the other side also.